Journal of Sound and Vibration (1998) **218**(1), 159–163
Article No. sv981770

LETTERS TO THE EDITOR

VIBRATIONS OF ORTHOTROPIC, CIRCULAR ANNULAR PLATES OF NON-UNIFORM THICKNESS AND A FREE INNER EDGE

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(*Received* 5 *June* 1998)

1. introduction

A rather limited amount of information is available in the case of vibrating circular plates of rectangular orthotropy in spite of its technological importance [1]. The situation is more critical in the case of annular plates and even more when the plate thickness is not uniform [2, 3].

The present study deals with the determination of the fundamental frequency of transverse vibration of the structural element shown in Figure 1. Two independent computational mechanics schemes are used: the optimized Rayleigh–Ritz method [4], and the finite element method making use of a very efficient code [5].

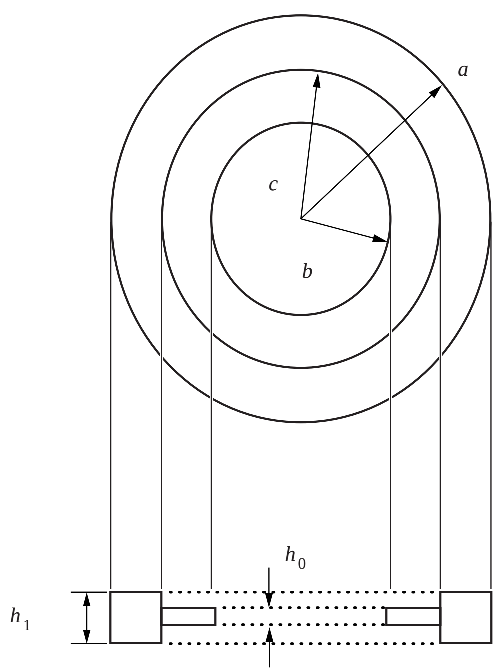

Figure 1. Vibrating system under study.

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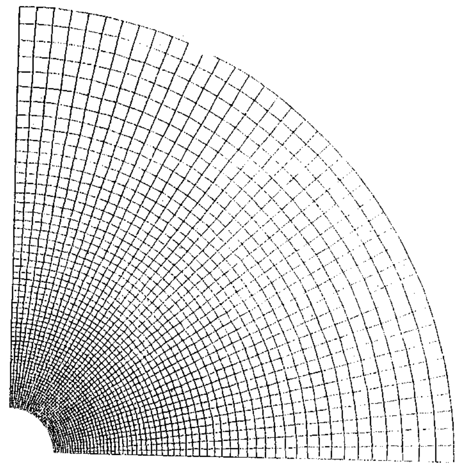
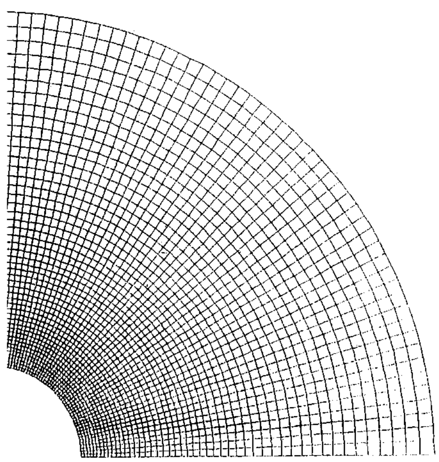


(a)



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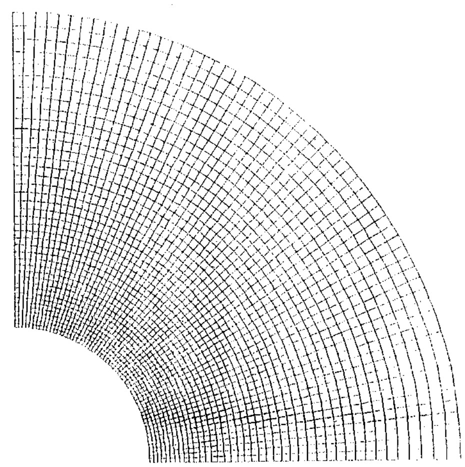

(c)

Figure 2. Finite element meshes: (a) $b/a = 0{\cdot}1$ (b) $b/a = 0{\cdot}$ (c) $b/a = 0{\cdot}3$.

2. APPROXIMATE ANALYTICAL SOLUTION

Using Leknitskii's classical notation [1] one expresses the governing functional in the form

$$\mathrm{J}[W] = \frac{1}{2}\iint\left[D_1\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + 2D_1\nu_2\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + D_2\left(\frac{\partial^2 W}{\partial y^2}\right)^2\right.$$

$$\left.+4D_k\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2\right)\mathrm{d}x\,\mathrm{d}y - \frac{\rho h}{2}\omega^2\iint W^2\,\mathrm{d}x\,\mathrm{d}y, \qquad (1)$$

where $W(x, y)$ is the amplitude of transverse vibrations. As shown in previous studies [2, 3] the fundamental mode shape is approximated by means of a polynomial of the form

$$W \simeq W_\alpha = A_1(\alpha r^\gamma + \beta r^3 + 1), \tag{2}$$

where α and β are obtained substituting equation (2) in the conditions

$$W(a) = 0, \qquad \left.\frac{d^2W}{dr^2} + \frac{v_2}{r}\frac{dW}{dr}\right|_{r=a} = 0 \tag{3a, b}$$

in the case of an outer, simply supported edge. Equation (3b) is an approximate requirement when the plate is rectangularly orthotropic but it is the exact boundary condition if the plate is isotropic and for this case $v_2 = v$.

When the annular plate is clamped at the outer edge one has two essential boundary conditions,

$$W(a) = \frac{dW}{dr}(a) = 0. \tag{4a, b}$$

TABLE 1

Fundamental frequency coefficients $\Omega_1 = \sqrt{\rho h / D_{(1)}}\,\omega_1 a^2$ of isotropic, annular plates of non-uniform thickness with a free inner edge

b/a	c/a	R–R $v = 1/3$	F.E. $v = 1/3$	R–R $v = 0{\cdot}3$	F.E. $v = 0{\cdot}3$
Isotropic					
Simply Supported					
0·1	0·2	4·8737	4·783	4·8352	4·753
	0·3	4·7425	4·680	4·7110	4·655
	0·4	4·6270	4·569	4·5993	4·547
	0·5	4·5226	4·455	4·4966	4·435
0·2	0·3	4·6644	4·586	4·6532	4·580
	0·4	4·5194	4·469	4·5139	4·467
	0·5	4·4047	4·357	4·4010	4·357
0·3	0·4	4·5479	4·507	4·5586	4·519
	0·5	4·4067	4·381	4·4211	4·397
Clamped					
0·1	0·2	10·1532	10·012	10·1773	10·050
	0·3	10·0514	9·955	10·0866	10·000
	0·4	10·0310	9·944	10·0715	9·994
	0·5	10·0470	9·960	10·0882	10·011
0·2	0·3	10·4010	10·293	10·4668	10·365
	0·4	10·3907	10·314	10·4637	10·392
	0·5	10·4122	10·348	10·4879	10·428
0·3	0·4	11·5171	11·451	11·6075	11·545
	0·5	11·6241	11·572	11·7193	11·670

R–R, optimized Rayleigh–Ritz results; F.E., finite element results.

TABLE 2

Fundamental frequency coefficients $\Omega_1 = \sqrt{\rho h/D_{1(1)}}\,\omega_1 a^2$ *of orthotropic, annular plates of non-uniform thickness with a free inner edge*

b/a	c/a	R–R $\nu_2 = 1/3$	F.E. $\nu_2 = 1/3$	R–R $\nu_2 = 0{\cdot}3$	F.E. $\nu_2 = 0{\cdot}3$
Orthotropic					
Simply Supported					
0·1	0·2	4·3931	4·303	4·3509	4·261
	0·3	4·2748	4·210	4·2403	4·171
	0·4	4·1707	4·110	4·1401	4·074
	0·5	4·0766	4·007	4·0476	3·973
0·2	0·3	4·2045	4·123	4·1925	4·099
	0·4	4·0737	4·018	4·0679	3·997
	0·5	3·9703	3·917	3·9663	3·898
0·3	0·4	4·0994	4·050	4·1115	4·041
	0·5	3·9721	3·937	3·9882	3·931
Clamped					
0·1	0·2	9·1520	9·013	9·1785	8·996
	0·3	9·0602	8·656	9·0990	8·748
	0·4	9·0422	8·725	9·0865	8·818
	0·5	9·0565	8·791	9·1018	8·884
0·2	0·3	9·3753	9·267	9·4478	9·275
	0·4	9·3660	9·287	9·4465	9·299
	0·5	9·3854	9·318	9·4689	9·332
0·3	0·4	10·3814	10·315	10·4811	10·333
	0·5	10·4778	10·425	10·5829	10·445

R–R, optimized Rayleigh–Ritz results; F.E., finite element results.

The exponential parameter γ appearing in equation (2) constitutes Rayleigh's optimization parameter [4].

3. FINITE ELEMENT SOLUTION AND RESULTS

One-quarter of the plate domain was subdivided into 2400 elements. Figure 2 depicts the corresponding meshes for (a) $b/a = 0{\cdot}1$ (b) $b/a = 0{\cdot}2$ and (c) $b/a = 0{\cdot}3$. As previously mentioned, ALGOR code [5] was used in the present analysis.

The fundamental frequency coefficients Ω_1 were referred to the thicker portion of the plate (thickness: h_1). When using the optimized Rayleigh–Ritz method the frequency coefficient Ω_1 was minimized with respect to γ.

All calculations were performed for $h_0/h_1 = 0{\cdot}8$ and, when dealing with an orthotropic material, the following constitutive properties were used: $D_2/D_1 = 1/2$; $D_k/D_1 = 1/3$.

In the case of an isotropic plate the frequencies were evaluated for Poisson's ratio equal to 0·3 and 1/3 and when dealing with the orthotropic structure ν_2 was also taken equal to 0·3 and 1/3.

Table 1 presents frequency coefficients Ω_1 for the isotropic plate while Table 2 deals with the orthotropic configuration.

The finite element results are extremely accurate in view of the high number of elements used and the analytical results are in very good engineering agreement with them as shown in Tables 1 and 2. The fact that a very simple polynomial approximation gives high accuracy in a rather complex elastodynamics problem is a remarkable fact. In general, the agreement is better in the case of plates simply supported at the outer edge. For $b/a = 0{\cdot}3$ and $c/a = 0{\cdot}4$ and $0{\cdot}5$ one observes a clear, dynamic stiffening effect.

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program and by Secretaría General de Ciencia y Tecnología of Universidad Nacional del Sur.

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